Burlington County Institute of Technology

Medford Campus

Westampton Campus

Geometry Curriculum

Department: Mathematics
Credits: 5
Revised: August 2023
Board Approval Date: August, 2023

Course Description

Geometry is a branch of mathematics that deals with the study of shapes, sizes, and properties of space. This course provides a comprehensive introduction to the fundamental principles of geometry, including points, lines, angles, planes, and curves.

Students will explore topics such as congruence, similarity, and symmetry, as well as the properties of polygons, circles, and other geometric figures. They will learn to use different methods of proof, including deductive reasoning and coordinate geometry, to solve problems related to geometric concepts.

The course also covers the relationships between geometry and other branches of mathematics, such as algebra and trigonometry. Students will learn to apply their knowledge of geometry to real-world situations, including architecture, visual arts, and science.

Throughout the course, students will develop their critical thinking and problem solving skills by working on a variety of challenging exercises and projects. They will also enhance their ability to communicate mathematical ideas clearly and effectively through writing and oral presentations.

By the end of this course, students will have a deep understanding of the principles of geometry and their applications, and will be able to apply their knowledge to a wide range of mathematical and practical problems.

Table of Contents

Geometry Pacing Guide

Unit	Standards	Days
Unit 1: Spatial Relations	G-GPE: 5, 7, G-MG: 3 G-GMD.3 G-CO: 1, 9, 12 G-SRT: 8	24.5
Unit 2: Congruency	G-CO: 2, 3, 4, 5, 6, 7, 8, 10, 11, 12 G-GMD: 4 G-GPE: 4, 7 G-MG: 1, 3 G-SRT: 5	26
Unit 3: Similarity and Transformations	G-CO: 2 G-MG: 1, 3 G-SRT: 1, 2, 3, 4, 5	10.5
Unit 4: Circles	G-C: 1, 2, 3, 5 G-CO: 1, 12, 13 G-GMD: 1 G-GPE: 1, 2, 6 G-MG: 3	8.5
Unit 5: Trigonometry	G.SRT: 4, 5, 6, 7, 8	7
Unit 6: Quadrilaterals	G-CO: 11 G-GPE: 4 G-MG: 3	4
Unit 7: Area and Volume	G-GMD: 1, 3, 4 G-MG: 1, 2, 3 S-MD: 7	2

Curriculum Maps

Unit 1: Spatial Relations (24.5 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Congruence (G-CO)
  - 🔷 Experiment with transformations in the plane (G-CO.1)
  - 🟩 Prove geometric theorems (G-CO.9).
  - 🔷 Make geometric constructions (G-CO.12)
- Similarity, Right Triangles, and Trigonometry (G-SRT)
  - 🟩 Solve problems involving right triangles (G-SRT.8).
- Expressing Geometric Properties with Equations (G-GPE)
  - 🟩 Use coordinates to prove simple geometric theorems algebraically (G-GPE.5, G-GPE.7).
- Geometric Measurement and Dimensions (G-GMD)
  - 🟡 Explain volume formulas and use them to solve problems (G-GMD.3)
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Comprehending and applying basic geometric terms are important for future theorems and postulates. ○ Mathematical notation and language is necessary for a strong foundation in Geometry. ○ Geometric relationships and definitions can be used to construct geometric figures and solve real world problems. ○ Algebraic properties of equality are used in geometry to solve problems and justify reasoning. ○ Practicing geometric proofs teaches the logic of deductive reasoning. ○ Geometric constructions are evident and necessary in the surrounding environment.	○ Why do you need to know, understand, and speak in geometric terms to function in everyday life? ○ Why is it important to prove your position in an argument (mathematically or in general)? ○ What is the significance of proper notation in the study of geometry? ○ Why do we measure? How does what we measure influence how we measure? ○ How are constructions different from sketches and drawings?
Students will know:	**Students will be able to:**
○ formulas can be used to find the midpoint and length of any segment in the coordinate plane. ○ formal geometric constructions can be created with a variety of tools and methods to provide a visual representation of geometric concepts. ○ some mathematical relationships can be described using a variety of if-then statements. ○ special angle pairs can be used to identify	○ identify and model points, lines, planes, intersecting lines, and intersecting planes. ○ measure segments and calculate with measurements. ○ find the distance between two points and the midpoint of a segment. ○ measure and classify angles. ○ identify and use congruent angles and the

geometric relationships and to find angle measures. ○ the special angle pairs formed by parallel lines and a transversal are either congruent or supplementary. ○ certain angle pairs can be used to determine whether two lines are parallel. ○ by comparing the slopes of two lines it can be determined whether the lines are parallel or perpendicular.	bisector of an angle. ○ identify and use special pairs of angles. ○ identify perpendicular lines. ○ identify and name polygons. ○ find perimeter, circumference, and area of two-dimensional figures. ○ identify and name three-dimensional figures. ○ find surface area and volume. ○ use the Pythagorean Theorem to calculate distance in the coordinate plane. ○ identify and use basic postulates about points, lines, and planes. ○ write proofs involving segment addition and segment congruence. ○ write proofs involving supplementary, complementary, congruent, and right angles. ○ identify the relationships between two lines or two planes. ○ name angle pairs formed by parallel lines and transversals. ○ use theorems to determine the relationships between specific pairs of angles. ○ use algebra to find angle measures. ○ recognize angles pairs that occur with parallel lines. ○ prove that two lines are parallel.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Real World Geometry Picture Dictionary: Create a photo/picture geometry dictionary containing a title page, 1 entry per term including definitions (in	**<u>Required District/State Assessments:</u>** ○ Unit Assessment ○ SGO Assessments ○ Proofs Portfolio: Collection of student proofs

your own words) and pictures taken by you of geometric figures found in the world around you. You may not use the same photo/picture to describe two different geometric terms/concepts. Put together a dictionary using Google Slides to be shared with the class for peer review. ○ Parallel Lines Cut by a Transversal Maze (Google Slides) [Available for download under Math Shared Folder > Geometry > 03- Parallel and Perpendicular Lines]	demonstrating mastery of utilizing theorems and postulates throughout the unit. **<u>Suggested Formative/Summative Assessments:</u>** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

<u>Learning Activities:</u>

- NJSLA Released Items
- Starter exercises
- Guided notes

- In class activities (matching, scavenger hunt, interactive exercises, etc.)
  - ⇒ Measuring with Accuracy
  - ⇒ As the Crow Flies
  - ⇒ Classifying Angles Round Table
  - ⇒ Name Grid
  - ⇒ Discover the Postulates Investigation
  - ⇒ I Have Who Has Vocabulary
  - ⇒ Constructed Response Practice
  - ⇒ Constructions
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

ELA Connection (RL.10.4) - Name of Tasks: Defining Parallel lines

- This task asks students to analyze definitions: Are they mathematically sound, complete, accurate, confusing? This challenges students to look at the concepts more closely and understand how important definitions are.
- Example: Alex and his friends are studying for a geometry test and one of the main topics covered is parallel lines in a plane. They each write down what they think it means for two distinct lines in a plane to be parallel: a. Rachel writes, ''two distinct lines are parallel when they are both perpendicular to a third line.'' b. Alex writes, ''two distinct lines are parallel when they do not meet.'' c. Briana writes, ''two distinct lines are parallel when they have the same slope.'' Analyze each definition, indicating if it is mathematically correct and if it has any drawbacks.
- Example: Three students have proposed these ways to describe when two lines l and m are perpendicular. Explain why each of these definitions is correct. What are some of the advantages and disadvantages with each?
  - ⇒ l and m are perpendicular if they meet at one point and one of the angles at the point of intersection is a right angle.
  - ⇒ l and m are perpendicular if they meet at one point and all four of the angles at their point of intersection are right angles.

⇒ l and m are perpendicular if they meet at one point and reflection about l maps m to itself

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Microsoft Excel can be used to create a 2-column formal proof of theorems.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
  - ⇒ Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

NJ SEL Competencies

- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals.
- Responsible Decision-Making: Develop, implement, and model effective problem solving and critical thinking skills.

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can deepen their understanding of proofs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.
- Model the thinking and processes involved in constructing a two column or paragraph proof.
- Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.
- Encourage students to use their reference notebook when constructing proofs as a tool.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions.
- Use this information to tailor instruction to student needs.

ELL:

- Create and explain orally the proofs of theorems in student's native language and/or use gestures, examples and selected technical words
- Provide graphic organizers
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students will extend their knowledge of points, lines, and planes in Euclidean geometry to the Fano Plane in which there is a finite number of lines and points in a plane called a projective plane.
  - ⇒ Students will extend their knowledge of angle measures in degrees to finding the complement and supplement of angle measures given in degrees, minutes, and seconds.
  - ⇒ Students will extend their knowledge of angle relationships to design a runway system with the fewest number of runways so that a pilot will never have a crosswind angle of more than 30 degrees. (Enrichment WS Runway Angles)
  - ⇒ Students will extend their knowledge of parallel lines and traversals to spherical geometry and longitude and latitude (Enrichment WS Spherical Geometry).

Unit 2: Congruency (26 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Congruence (G-CO)
  - 🔷 Experiment with transformations in the plane (G-CO.2, G-CO.3, G-CO.4, G-CO.5)
  - 🟩 Understand congruence in terms of rigid motion (G-CO.6, G-CO.7, G-CO.8).
  - 🟩 Prove geometric theorems (G-CO.10, G-CO.11).
  - 🔷 Make geometric constructions (G-CO.12).
- Similarity, Right Triangles, and Trigonometry (G-SRT)
  - 🟩 Prove theorems involving similarity (G-SRT.5).
- Expressing Geometric Properties with Equations (G-GPE)
  - 🟩 Use coordinates to prove simple geometric theorems algebraically (G-GPE.4, G-GPE.7).
- Geometric Measurement and Dimensions (G-GMD)
  - 🟡 Visualize relationships between two-dimensional and three-dimensional objects (G-GMD.4).
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.1, G-MG.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Transformations can be applied in real-life situations. ○ Rigid transformations preserve size and shape or distance and angles, within the concept of congruency. ○ Classifying helps to develop connections among mathematical ideas. Coordinate geometry can be used to prove general relationships and to classify polygons. ○ Properties of geometric figures can be proven.	○ How can geometric properties be used to prove relationships between the angles and sides of geometric figures? ○ How does a transformation change the image of a figure? ○ What types of motion in the plane maintain the congruence of a figure?
Students will know:	**Students will be able to:**
○ minimal information is needed to prove triangles congruent. ○ a triangle can only exist if the dimensions fall within a certain range. ○ the sum of the interior angles of any triangle is 180 degrees. ○ figures are congruent if and only if their corresponding parts are congruent. ○ the sum of the interior angle measures of a polygon depends on the number of sides the polygon has. ○ polygons are identified by the number of their sides. ○ the angles and sides of isosceles and equilateral	○ identify and classify triangles by angle and side measures. ○ apply the Triangle Angle-Sum Theorem and Exterior Angle Theorem. ○ name and use corresponding parts of congruent polygons. ○ prove triangles congruent using the definition of congruence. ○ use the SSS, SAS, ASA, and AAS to test for triangle congruence. ○ use properties of isosceles and equilateral triangles. ○ find and use the sum of the measures of the interior and exterior angles of a polygon.

triangles have special relationships. ○ special segments of a triangle meet at a point of concurrency. ○ formulas for midpoint, length, and slope can be used to verify specific relationships in the coordinate plane. ○ the angles, sides and diagonals of parallelograms have special relationships. ○ there are different types of congruence transformations. ○ symmetry, both line and rotational, appear in nature and in art.	○ recognize and apply properties of the sides, angles, and diagonals of parallelograms. ○ recognize the conditions that ensure a quadrilateral is a parallelogram. ○ prove that a set of points forms a parallelogram in the coordinate planes. ○ find perimeters and areas of triangles and parallelograms. ○ identify and use perpendicular bisectors, angle bisectors, medians, and altitudes in triangles. ○ recognize and apply properties of inequalities to the measure of the angles of a triangle and to the relationships between the angles and sides of a triangle.. ○ use the Triangle Inequality Theorem to identify possible triangles and prove triangle relationships. ○ identify and draw reflections, translations, and rotations in the coordinate plane. ○ verify congruence after a congruence transformation. ○ draw glide reflections and other compositions of isometries in the coordinate plane. ○ draw compositions of reflections in parallel and intersecting lines. ○ identify line and rotational symmetries in two-dimensional figures and plane and axis symmetries in three-dimensional figures.
Assessment Evidence	
Suggested Performance Tasks: ○ Congruent Triangle Logo Project [Available for download under Math Shared Folder > Geometry	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments

> 04- Congruent Triangles] ○ Standard Solutions Transformations PAT [Available for download under Math Shared Folder > Geometry > 05- Parallel and Perpendicular Lines]	○ Proofs Portfolio: Collection of student proofs demonstrating mastery of utilizing theorems and postulates throughout the unit. **<u>Suggested Formative/Summative Assessments:</u>** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

<u>Learning Activities:</u>

○ NJSLA Released Items
○ Starter exercises

- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
    - ⇒ Classifying Triangles Activity
    - ⇒ Discover the Exterior Angle Theorem
    - ⇒ Discover the Triangle Angle Sum Theorem
    - ⇒ Constructions:
    - ⇒ Patty paper: Show whether SSS, SAS, and SSA work for triangle congruency
    - ⇒ Patty paper: Showing whether AAS and AAA work for triangle congruency
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Textbook:

Carter, John, et al, Geometry, Glencoe, McGraw-Hill, 2010 and 2012

Related Standards

Interdisciplinary connections

ELA Connection:

- SL.9-10.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any false reasoning or distorted evidence.
    - ⇒ Example: Students will need to be able to read, write, and follow the sequence of a proof.

Technology (NJSLS Computer Science & Design Thinking)

- 8.2.8.ED.2: Identify the steps in the design process that could be used to solve a problem.
    - ⇒ Example: Students can create an MC Esher-like tessellation using Geogebra, explore its properties, and present their findings in Google Slides.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.8.TL.1: Construct a spreadsheet in order to analyze multiple data sets, identify relationships, and facilitate data-based decision-making.
  - ⇒ Example: Microsoft Excel can be used to create a 2-column formal proof of theorems.
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: Students can use dynamic computer programs, such as Geogebra, to investigate the properties of transformations, such as the effect of reflecting an object over parallel lines. Geogebra allows students to set parameters and then manipulate polygons according to those parameters so that students can inductively reason through complex geometric ideas and discover geometric properties on their own.

NJ SEL Competencies

- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges
- Self-Management: Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can deepen their understanding of proofs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build

understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Model the thinking and processes involved in constructing a two column or paragraph proof involving triangles. Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions.
- Use this information to tailor instruction to student needs.
- Pre-teach triangle congruence criteria using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Provide students with opportunities to practice applying these concepts and skills by working in pairs/small groups.
- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Allow time for questions to check on student understanding.
- Review thinking and processes involved in

ELL:

- Create and explain orally the proofs of theorems in a student's native language and/or use gestures, examples and selected technical words.
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for.
- Supply a translation dictionary.
- Use sentence stems to provide additional language support.
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem or explain their proof work.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition.

constructing a two column or paragraph proof. ○ Review converse statements. ○ Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency. ○ Encourage students to use their reference notebook as a tool when constructing proofs. ○ Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions. ○ Illustrate the congruence properties. Draw the similarities between these properties and the properties of equality. ○ Review congruence transformations that preserve length- translations, rotations, and reflections. ○ Explain the use of tick marks to show congruency. Be sure that students understand that they may see tic marks on other figures as well. ○ Review symbols for parallel and congruent. Review the definitions of perpendicular and bisector. ○ Encourage students to justify their reasoning. Provide students with sentence stems if needed. ○ Anchor charts to model strategies and processes. ○ Reference sheets that list formulas, step-by-step procedures and model strategies. ○ Conceptual word wall that contains definition, translation, pictures and/or examples. ○ Graphic organizers to help students model geometric properties. ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors. ○ Create an interactive notebook with students with	○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Prompt students to use an organizational chart to answer multi-step or multi-part word problems. ○ Sequence and explain the steps to prove theorems about triangles in a student's native language and/or use gestures, examples and selected technical words. ○ Provide students with graphic organizers. ○ Explain orally and in writing how to use congruence criteria for triangles to prove relationships in a student's native language and/or use drawings, examples and selected technical words. ○ Teacher Modeling ○ Use of word wall. ○ Use of graphic organizers.

a table of contents so they can refer to previously taught material readily. ○ Videos to reinforce skills and thinking behind concepts. ○ Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems.	

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can use coordinate point descriptions of triangles to classify the triangle (4.1 Enrichment: Reading Mathematics)
  - ⇒ Students can explore angles and sides in overlapping congruent triangles (4.3 Enrichment: Overlapping Triangles)
  - ⇒ Students can determine if two triangles are congruent by using coordinate points (4.4 Enrichment: Congruent Triangles in the Coordinate Plane)
  - ⇒ Students can prove that the Angle Bisector construction from Unit 1 will create congruent triangles and, therefore, must bisect the angle (4.4 Geometry Lab: Proving Constructions)
  - ⇒ Students can use dynamic geometry programs to test SSA for right triangles (4.5 Geometry Lab: Congruence in Right Triangles)
  - ⇒ Students can investigate whether or not AAA is a valid triangle congruence test (4.5 Enrichment: What about AAA to Prove Triangle Congruence?)
  - ⇒ Students can inscribe and circumscribed triangles (5.1 Enrichment: Inscribed and Circumscribed Triangles)
  - ⇒ Students can construct medians and altitudes and use them to find centroids and orthocenters, respectively (5.2 Geometry Lab: Constructing Medians and Altitudes and 5.2 Enrichment: Constructing Centroids and Orthocenters)
  - ⇒ Students can use dynamic geometry programs to discover the Triangle Inequality Theorem (5.5 Graphing Technology Lab: The Triangle Inequality)
  - ⇒ Students can construct triangles or determine if the triangles are impossible to construct (5.5 Enrichment: Constructing Triangles)
  - ⇒ Students can use dynamic geometric programs and/or patty paper to determine properties of

rotations, reflections, and translations, both in space and on the coordinate plane.

⇒ Students can explore Solids of Revolution by rotating a plane figure or curve about a line (9.3 Geometry Lab: Solids of Revolution)

⇒ Students can use dynamic geometry programs to discover the effects of performing multiple transformations on a figure (9.4 Geometry Software Lab: Compositions of Transformations).

⇒ Students can use a ruler and protractor to scale images given a scale factor (9.4 Enrichment: Computer Graphics)

⇒ Students can explore and create many types of tessellations (9.4 Geometry Lab: Tessellations)

⇒ Students can explore symmetry in different quadrilaterals (9.5 Enrichment: Symmetry in Quadrilaterals)

Unit 3: Similarity and Transformations (10.5 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Congruence (G-CO)
  - 🔷 Experiment with transformation in the plane (G-CO.2)
- Similarity, Right Triangles, and Trigonometry (G-SRT)
  - 🟩 Understand similarity in terms of similarity transformations (G-SRT.1, G-SRT.2, G-SRT.3).
  - 🟩 Prove theorems involving similarity (G-SRT.4, G-SRT.5).
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.1, G-MG.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**

○ Scale provides a mechanism by which we can solve problems related to large or small real world objects. This scale represents a constant ratio between corresponding lengths of sides of similar figures. ○ Proportional relationships express how quantities change in relation to each other. ○ An object in a plane can be oriented in an infinite number of ways while maintaining its shape.	○ How do you recognize similarity in figures? ○ What are the effects of performing dilations on geometric figures? ○ How are transformations and similar figures related? ○ To what extent do professionals use scale to analyze real world extremely large or extremely small objects? ○ How does the constant of a proportion affect a figure and how is the constant determined? ○ How can transformations be described mathematically?
Students will know: ○ similar polygons have proportional corresponding medians, bisectors, and altitudes. ○ minimal information is needed to prove triangles are similar. ○ Proportional equations are useful in solving problems in a variety of applications. figures are similar if and only if their corresponding sides are proportional and corresponding angles are congruent. ○ dilations are similarity transformations.	**Students will be able to:** ○ use proportions to identify similar polygons. ○ solve problems using the properties of similar polygons. ○ identify similar triangles using the AA Similarity Postulate and the SSS and SAS Similarity Theorems. ○ use similar triangles to solve problems. ○ use proportional parts within triangles and with parallel lines. ○ recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles. ○ use the Triangle Bisector Theorem. ○ identify and verify similarity transformations. ○ draw dilations in the coordinate plane. ○ find areas of similar figures by using scale factors. ○ find scale factors or missing measures given the areas of similar figures.

Assessment Evidence

Suggested Performance Tasks:

- Find the Height of a Flagpole using Similar Triangles Project [Available for download under Math Shared Folder > Geometry > 07- Proportions and Similarity]
- MAP Puzzling Triangles Assessment Task [Available for download under Math Shared Folder > Geometry > 07- Proportions and Similarity]

Required District/State Assessments:

- Unit Assessment
- SGO Assessments
- Proofs Portfolio: Collection of student proofs demonstrating mastery of utilizing theorems and postulates throughout the unit.

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (proofs matching, scavenger hunts, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science & Mathematics Career Pathway Connection:

- ST-SM; 9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
  - ⇒ Example: Students can indirectly measure the height of a flagpole using similarity rules.

Visual Arts Connection:

- 1.5.12prof.Re7a: Hypothesize ways in which art influences perception and understanding of human experiences.
  - ⇒ Example: Introduce artwork from MC Escher in order to discuss how geometric properties are represented in art. Specifically show pictures of impossible drawings.

Physical Education Connection:

- 2.2.2.MSC.5: Adjust and correct movements and skill in response to feedback.
  - ⇒ Backyard Basketball task using similar triangles to calculate the height of a basketball net. Discuss how the height of the basketball net influences the manipulative movements of throwing the ball.

Technology (NJSLS Computer Science & Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
    - ⇒ Example: Students can use dynamic computer programs, such as Geogebra, to investigate the properties of dilations, such as the scale factor of the perimeter and area of two similar figures. Geogebra allows students to set parameters and then manipulate polygons according to those parameters so that students can inductively reason through complex geometric ideas and discover geometric properties on their own.
    - ⇒ Example: Students can digitally create dilations that verify the concept that corresponding sides from a pre-image to an image are parallel and length is affected by the scale factor utilizing NCTM Isometric Drawing tool.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
    - ⇒ Example: Students will apply previously acquired skills in writing ratios and solving proportions to determine whether or not polygons are similar and to find missing sides in real-world as well as abstract problems.
    - ⇒ Example: Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create similarity transformations to determine the similarity of figures and explain the meaning of similarity for triangles. Students will also ask probing questions to clarify and improve arguments.
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
    - ⇒ Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

NJ SEL Competencies

- Social Awareness: Demonstrate an awareness of the expectations for social interactions in a variety of settings
- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking

skills

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can deepen their understanding of solving for missing parts of similar polygons by presenting multiple ways to solve the same problem.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Model the thinking and processes involved in constructing a two column or paragraph proof.

ELL:

- Create and explain orally the proofs of theorems in a student's native language and/or use gestures, examples and selected technical words.
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best

○ Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency. ○ Encourage students to use their reference notebook when constructing proofs as a tool. ○ Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions. ○ Use this information to tailor instruction to student needs. ○ Encourage students to justify their reasoning. Provide students with sentence stems if needed. ○ Anchor charts to model strategies and processes. ○ Reference sheets that list formulas, step-by-step procedures and model strategies. ○ Conceptual word wall that contains definition, translation, pictures and/or examples. ○ Graphic organizers to help students model geometric properties. ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors. ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily. ○ Videos to reinforce skills and thinking behind concepts. ○ Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems. ○ Explain the Cross Multiplication Theorem as something that the students already know from previous classes about how to solve a proportion. This is a formal way of writing it. ○ Intention is to explore relationships between	when they have defined roles (surrounding the content they are studying) that they are responsible for. ○ Supply a translation dictionary. ○ Use sentence stems to provide additional language support. ○ Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills. ○ The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their proof work. ○ Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit). ○ Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Prompt students to use an organizational chart to

proportional side lengths and congruent angles of similar triangles. ○ Review ways to figure out if two triangles are similar using side lengths and angles. ○ Review how to calculate the area of common figures (triangles, parallelograms, circles, etc). ○ Introduce vocabulary of “scale factor of dilation” and “similarity ratio” (not scale factor of similarity).	answer multi-step or multi-part word problems.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can extend their understanding of similar triangles by completing a hands-on project involving the height of a flagpole and shadow length.
  - ⇒ Students can construct similar polygons, either using a compass and straightedge or dynamic geometry programs.
  - ⇒ Students can use similar triangles to prove that the slopes of perpendicular lines are negative reciprocals and the slopes of parallel lines are equal (7.3 Geometry Lab: Proofs of Perpendicular and Parallel Lines).
  - ⇒ Students can use similar triangles and proportions to prove the Pythagorean Theorem for right triangles (7.5 Enrichment: A Proof of the Pythagorean Theorem).
  - ⇒ Students can use a compass and straightedge to create medial and orthic triangles (7.6 Enrichment: Medial and Orthic Triangles).
  - ⇒ Students can use dynamic geometry programs to explore properties of dilations (9.6 Graphing Technology Lab: Dilations)
  - ⇒ Students can map circles onto one another using dilations and find a center of dilation (9.6 Enrichment: Similar Circles)

Unit 4: Circles (8.5 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Congruence (G-CO)
  - 🔷 Experiment with transformations (G-CO.1).
  - 🔷 Make geometric constructions (G-CO.12, G-CO.13).
- Circles (G-C)
  - 🟡 Understand and apply theorems about circles (G-C.1, G-C.2, G-C.3).
  - 🟡 Find arc lengths and areas of sectors of circles (G-C.5)
- Expressing Geometric Properties with Equations (G-GPE)
  - 🟩 Use coordinates to prove simple geometric theorems (G-GPE.6).
  - 🟡 Translate between the geometric description and the equation for a conic section (G-GPE.1, G-GPE.2)
- Geometric Measurement and Dimensions (G-GMD)
  - 🟡 Explain volume formulas and use them to solve problems (G-GMD.1)
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ A circle is a unique geometric shape that appears in nature and in everyday objects. ○ There is a specific relationship between the circumference and the diameter of a circle. ○ Relationships exist between angles and arc measurements.	○ Why are circles special as a geometric shape? ○ What is the relationship between the circumference and the diameter of a circle? ○ How do we know which method to use when solving problems involving circles? ○ How do we use circles to model and solve real world situations? ○ When solving real world problems, what assumptions have to be made? ○ How is the equation of a circle derived given the radius and the center coordinates?
Students will know:	**Students will be able to:**
○ circumference and area can be determined from the diameter of a circle. ○ relationships exist among the many segments, angles and arcs related to a circle. ○ pi is an irrational number that was discovered thousands of years ago and is still being studied today.	○ identify and use parts of circles. ○ solve problems involving the circumference of a circle. ○ identify central angles, major arcs, minor arcs, and semicircles, and find their measures. ○ find arc lengths. ○ recognize and use relationships between arcs, chords, and diameters. ○ find measures of inscribed angles and angles of inscribed polygons. ○ use properties of tangents. ○ solve problems involving circumscribed polygons.

	○ find areas of circles and sectors of circles.

Assessment Evidence

Suggested Performance Tasks:

- Merry Go Round PBA [Available for download under Math Shared Folder > Geometry > 10-Circles]
- RADIO STATIONS: Seven new radio stations must be assigned broadcast frequencies. tha stations are located at A(9,2), B(8,4), C(8,1), D(6,3), E(4,0), F(3,6), and G(4,5), where 1 unit = 50 miles. If stations that are more than 200 miles apart can share the same frequency, what is the least number of frequencies that can be assigned to these stations? Describe two different beginning approaches to solving this problem. Choose an approach, solve the problem, and explain your reasoning.

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Visual Arts Connection:

- 1.5.12prof.Re7a: Hypothesize ways in which art influences perception and understanding of human experiences.
  - ⇒ Example: Students recognize the presence of geometry in their real world by representing something tangible that interests them and modeling its geometric properties.

Technology (NJSLS Computer Science & Design Thinking)

- 8.2.12.ED.6: Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).
  - ⇒ Example: Students can compare the limitations of measuring pi using measurement tools, such as string and a ruler, against using a computer and investigate how our understanding of pi has changed over time.
  - ⇒ Example: Students can compare the limitation of using a compass and paper against using dynamic computer programs, such as Geogebra, to investigate circle properties.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
  - ⇒ Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
  - ⇒ Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will work in small groups to determine the value of pi by working together to measure the circumference and diameter of several shapes.

NJ SEL Competencies

- Relationship skills: Identify who, when, where, or how to seek help for oneself or others when needed.
- Responsible Decision-Making: Develop, implement, and model effective problem-solving and critical thinking skills.

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and

potential projects. Example: Students can explore properties of circles and explain their findings in small groups.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.
- Encourage students to use their reference notebook when constructing proofs as a tool.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions.
- Use this information to tailor instruction to student needs.
- Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Anchor charts to model strategies and processes.
- Reference sheets that list formulas, step-by-step procedures and model strategies.
- Conceptual word wall that contains definition, translation, pictures and/or examples.

ELL:

- Create and explain orally the proofs of theorems in a student's native language and/or use gestures, examples and selected technical words.
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for.
- Supply a translation dictionary.
- Use sentence stems to provide additional language support.
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem or explain their proof work.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).

○ Graphic organizers to help students model geometric properties. ○ Teacher modeling. ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily. ○ Videos to reinforce skills and thinking behind concepts. ○ Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems. ○ Demonstrate pi as the ratio of circumference to diameter for all circles and use this ratio to determine an equation for circumference of any circle. ○ Relate the pythagorean theorem to the equation of a circle. ○ Two circles are congruent if they have the same radius. Two circles are similar if they have different radii. Their similarity is shown through a ratio. ○ Define semicircles with a diagram, as well as major arcs and minor arcs. ○ Inscribed angles- vertex on circles, sides are chords, intercepts an arc of the circle. ○ When working on the multi-step examples, use color to help students to differentiate between which angles are being worked with and which ones aren't being worked with. The color will help students to focus on the appropriate part of the diagram.	○ Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Prompt students to use an organizational chart to answer multi-step or multi-part word problems.
Enrichment	
○ Challenge problems from resource sets	

- Extended learning goals:
  - ⇒ Students can extend their knowledge of parallel lines and traversals to spherical geometry and longitude and latitude (Enrichment WS Spherical Geometry).
  - ⇒ Students can use completing the square to determine the equation of a circle in standard form.
  - ⇒ Students can use a compass and straightedge or dynamic geometry programs to construct a circle given three noncollinear points.
  - ⇒ Students can use a compass and straightedge or dynamic geometry programs to construct a line tangent to a circle through an external point.
  - ⇒ Students can explore properties of the Reuleaux triangle and other curves of constant width (10.2 Enrichment: Curves of Constant Width).
  - ⇒ Students can explore patterns in chords of a circle (10.3 Enrichment: Patterns from Chords).
  - ⇒ Students will identify conic sections, translate between the geometric description of the equation for a parabola, and determine intersections between lines and parabolas in cooperative groups (Extend 10-8).

Unit 5: Trigonometry (7 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Similarity, Right Triangles, and Trigonometry (G-SRT)
  - 🟩 Prove theorems involving similarity (G-SRT.4, G-SRT.5).
  - 🟩 Define trigonometric ratios and solve problems involving right triangles (G-SRT.6, G-SRT.7, G-SRT.8).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Patterns exist in triangles. ○ Trigonometry can be used to solve various types of problems.	○ How do we use right triangles to model and solve real world situations? ○ How are trigonometric ratios defined?

	○ How do we measure a right triangle? ○ What is the relationship between cosine and sine in relation to complementary angles?
Students will know: ○ there are special relationships between parts of a right triangle and the altitude to its hypotenuse. ○ There are common triples that make up the sides of right triangles. ○ the types of angles in a triangle and thus the type of triangle can be determined using the converse of the Pythagorean Theorem. ○ certain right triangles have properties that allow their side lengths to be determined by applying the known rule (without using the Pythagorean Theorem) if you have the lengths of two sides of a right triangle, you can find the third by applying the Pythagorean Theorem. ○ The angles of elevation and depression are the acute angles of a right triangle formed by a horizontal distance and a vertical height. ○ trigonometric relationships exist between specific angles and sides of triangles.	**Students will be able to:** ○ find the geometric mean between two numbers. ○ solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse. ○ use the properties of 45°-45°-90°and 30°-60°-90° triangles. ○ find trigonometric ratios using right triangles and use trigonometric ratios to find angle measures in right triangles. ○ solve problems involving angles of elevation and depression and use angles of elevation and depression to find distance between two objects.
Assessment Evidence	
Suggested Performance Tasks: ○ How high is it? Project [Available for download under Math Shared Folder > Geometry > 08- Right Triangles and Trigonometry] ○ Standard Solutions Telephone Pole PAT [Available	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:**

for download under Math Shared Folder > Geometry > 08- Right Triangles and Trigonometry]	○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
  - ⇒ In-class journal to include real world examples and class discussion topics
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)

- Homework relating to current topic

Related Standards

Interdisciplinary connections

Career and Technical Education Architecture & Construction Connections:

- 9.3.12.AC.1 Use vocabulary, symbols and formulas common to architecture and construction.
  - ⇒ Example: Students can apply trigonometry to find angles of elevation and depressions as well as heights of buildings.
- 9.3.12.AC-DES.1 Justify design solutions through the use of research documentation and analysis of data.
  - ⇒ Example: You have been commissioned to design an access ramp, which complies with the Americans with Disabilities Act (ADA) requirements, for an entry that is 3 feet above ground level. Your client has asked you to design the ramp and to determine costs, using local pricing, for two types of ramps, wooden and concrete.

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.
  - ⇒ Example: Students can use paper and a ruler to determine trigonometric ratios and can then lend that understanding to the use of computer software to find the same ratios on similar triangles. Lastly, students can compare these calculations to the trigonometric ratios stored in their calculators, and use the calculator to determine missing lengths of triangles that may be difficult or impossible to draw. Through this process, students can understand the need for the stored trigonometric functions of their calculator.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
  - ⇒ Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon

solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will work in pairs or small groups to apply angles of elevation and depression and trigonometry in real world problems.

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

NJ SEL Competencies

- Relationship Skills: Identify who, when, where, or how to seek help for oneself or others when needed
- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can assist one another in solving applied trigonometry problems in small groups. Students can choose objects to indirectly measure using trigonometry.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations,

gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.
- Encourage students to use their reference notebook when constructing real world examples solved using trigonometric ratios.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions.
- Use this information to tailor instruction to student needs.Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Anchor charts to model strategies and processes.
- Reference sheets that list formulas, step-by-step procedures and model strategies.
- Conceptual word wall that contains definition, translation, pictures and/or examples.
- Graphic organizers to help students model geometric properties.
- Teacher modeling.

ELL:

- Create and explain orally the proofs of theorems in a student's native language and/or use gestures, examples and selected technical words.
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for.
- Supply a translation dictionary.
- Use sentence stems to provide additional language support.
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem or explain their proof work.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Identify key phrases or new vocabulary to

- Highlight and label solution steps for multi-step problems in different colors.
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily.
- Videos to reinforce skills and thinking behind concepts.
- Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems.
- Be sure that students understand an inverse of an operation undoes the operation.
- Remind students that the x in the cosine and sine ratios refers to the angle that we are focusing on.
- Apply trigonometry to indirect measures to demonstrate real world applications.
- Determine the angle of elevation and angle of depression for real-world problems; Use what you know about triangles and trigonometric ratios to model the problem.

pre-teach.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.
- Prompt students to use an organizational chart to answer multi-step or multi-part word problems.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Use similar triangles and the pythagorean theorem to discover geometric mean theorems. Apply these theorems to solve problems.
  - ⇒ Students can use dynamic geometry programs to explore trigonometric ratios (8.4 Graphing Technology Lab: Trigonometry)
  - ⇒ Students can calculate sec, csc, and cot trigonometric values (8.4 Graphing Technology Lab: Secant, Cosecant, and Cotangent).
  - ⇒ Students can explore sine and cosine values on the first quadrant of the unit circle (8.4 Enrichment: Sine and Cosine of Angles).
  - ⇒ Students can use the Pythagorean Theorem to construct values of square roots (8.3 Enrichment:

Constructing Values of Square Roots)

Unit 6: Quadrilaterals (4 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Congruence (G-CO)
  - 🟩 Prove geometric theorems (G-CO.11).
- Expressing Geometric Properties with Equations (G-GPE)
  - 🟩 Use coordinates to prove simple geometric theorems algebraically (G-GPE.4).
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.3)

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**

○ There are different ways to measure different dimensional figures. ○ The terms characteristics and properties can be used interchangeably to describe quadrilaterals. The term characteristics is used in elementary and middle school mathematics. ○ Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals. ○ Characteristics of quadrilaterals can be used to identify the quadrilateral and to find the measures of sides and angles.	○ What determines the classification of quadrilaterals? ○ How are the formulas for linear measure, area, and volume related to each other? ○ How can one find the area and/or perimeter of a figure composed of various basic geometric shapes? ○ How does what we measure influence how we measure?
Students will know: ○ a rectangle, rhombus, and square have all the properties of a parallelogram. A square has all the properties of a parallelogram, rectangle, and a rhombus. ○ diagonals of a rectangle are congruent and bisect each other. All four angles are right angles. ○ All sides of a rhombus are congruent, diagonals are perpendicular, and each diagonal bisects a pair of opposite angles. ○ In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.	**Students will be able to:** ○ recognize and apply properties of rectangles. ○ determine whether parallelograms are rectangles. ○ recognize and apply the properties of rhombi and squares. ○ determine whether parallelograms are rectangles, rhombi or squares. ○ apply properties of trapezoids and kites. ○ find areas of trapezoids, rhombi, kites, regular polygons, and composite figures.
Assessment Evidence	
Suggested Performance Tasks: ○ Describe the properties a quadrilateral must possess in order for the quadrilateral to be	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments

classified as rectangle, rhombus, square, trapezoid, isosceles trapezoid, or a kite. Compare and contrast the properties of all quadrilaterals.	**Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
    - ⇒ Create a Venn diagram that incorporates all quadrilaterals, including trapezoids, isosceles trapezoids,

kites, and quadrilaterals that cannot be classified as anything other than quadrilaterals.
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Career and Technical Education - Architecture & Design:

- 9.3.12.AC-DES.6 Apply the techniques and skills of modern drafting, design, engineering and construction to projects.
  - ⇒ Example: Students can use the properties of quadrilaterals to accurately lay out a square in a room (to represent furniture placement).

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
  - ⇒ Example: Students will become proficient in geometric technologies, such as Geogebra or Desmos. These digital tools allows students to set parameters and then manipulate polygons according to those parameters so that students can inductively reason through complex geometric ideas and discover geometric properties on their own.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

⇒ Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will demonstrate and explain to a peer or small group the criteria that can be used to determine whether or not a parallelogram belongs in the rectangle, rhombus, or square family.

NJ SEL Competencies

- Self-Management: Understand and practice strategies for managing one's own emotions, thoughts and behaviors
- Social Awareness: Recognize and identify the thoughts, feelings and perspectives of others

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can share their findings within small groups to inductively determine quadrilateral properties.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple

meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.
- Encourage students to use their reference notebook when constructing proofs as a tool.
- Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions.
- Use this information to tailor instruction to student needs.
- Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Anchor charts to model strategies and processes.
- Reference sheets that list formulas, step-by-step procedures and model strategies.
- Conceptual word wall that contains definition, translation, pictures and/or examples.
- Graphic organizers to help students model geometric properties.
- Teacher modeling.
- Highlight and label solution steps for multi-step problems in different colors.

ELL:

- Create and explain orally the proofs of theorems in a student's native language and/or use gestures, examples and selected technical words.
- Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for.
- Supply a translation dictionary.
- Use sentence stems to provide additional language support.
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem or explain their proof work.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Identify key phrases or new vocabulary to pre-teach.

- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily.
- Videos to reinforce skills and thinking behind concepts.
- Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems.
- Use the sum of the angles of a triangle to determine how to find the sum of the angles of any polygon. Demonstrate how to convert the pattern to a formula.
- List out the descriptions of a parallelogram.

- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.
- Prompt students to use an organizational chart to answer multi-step or multi-part word problems.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Given tape and string, students can tape off a SQUARE 8x8 area in the classroom.
  - ⇒ Students can maximize an area for an “animal pen” given a perimeter of fencing (6.4 Enrichment: Constant Perimeter)
  - ⇒ Students can construct Pythagorean Puzzles in order to demonstrate the Pythagorean Theorem (6.5 Enrichment: Creating Pythagorean Puzzles)
  - ⇒ Students can use dynamic geometry programs to calculate the areas of Trapezoids, Rhombi, and Kites and use their findings to inductively determine a general area formula (11.2 Graphing Technology Lab: Areas of Trapezoids, Rhombi, and Kites).
  - ⇒ Students can calculate population density of major cities and compare them to more rural areas (11.2 Geometry Lab: Population Density).

Unit 7: Area and Volume (2 days)

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Geometry

- Geometric Measurement and Dimensions (G-GMD)
  - 🟡 Explain volume formulas and use them to solve problems (G-GMD.1, G-GMD.3).
  - 🟡 Visualize relationships between two-dimensional and three-dimensional objects (G-GMD.4).
- Modeling with Geometry (G-MG)
  - 🟩 Apply geometric concepts in modeling situations (G-MG.1, G-MG.2, G-MG.3).

Statistics and Probability

- Using Probability to Make Decisions (S-MD)
  - \+ Use probability to evaluate outcomes of decisions (S-MD.7).

Enduring Understandings:	**Essential Questions:**
○ There are different ways to measure different dimensional figures.	○ How are the formulas for linear measure, area, and volume related to each other? ○ How can one find the area and/or perimeter of a figure composed of various basic geometric shapes? ○ How does what we measure influence how we measure?

Students will know:	**Students will be able to:**
○ units of measure differ when calculating linear measure, area, and volume. ○ formulas for area and volume exist and can be derived. ○ area and volume can be calculated using the appropriate formula. ○ a relationship exists among the scale factor, areas, and volumes of similar figures.	○ find areas of regular polygons and composite figures. ○ find lateral areas and surface areas of prisms, cylinders, pyramids, and cones. ○ find volumes of prisms, cylinders, pyramids, and cones. ○ find surface areas and volumes of spheres. ○ find probabilities by using length and area.

Assessment Evidence

Suggested Performance Tasks:

- Explain the difference between the lateral area and the surface area of a prism.
- Write a practical application problem involving the surface area or lateral area of a solid figure studied in this chapter.
- Give the dimensions of two cylinders in which the first has a greater volume than the second, but the second has greater surface area than the first.
- Draw and label the dimensions of a prism and a pyramid that have the same volume.

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio

	○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
  - ⇒ Name the Math activity
  - ⇒ Isometric Graph Paper
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Career and Technical Education - Manufacturing:

- 9.3.MN-QA.2 Recommend and implement continuous improvement in manufacturing processes.
  - ⇒ Example: Students can compare multiple containers and determine which has the most volume with the least amount of materials used.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
  - ⇒ Example: Students will use graphing calculators or spreadsheet technology to investigate geometric probability and the relationship between experimental and theoretical probabilities.
  - ⇒ Examples: Students can use graphing calculators or spreadsheet technology to investigate the probability of a geometric "game"; i.e. dropping a dart randomly onto a platform that is divided into different shapes. Students can hypothesize what will happen and use the program to test their theory, extending their data collection to hundreds of trials to use their experimental probability to closely approximate the theoretical probability. Students can then use geometric properties to calculate the actual theoretical probability and compare their results.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
  - ⇒ Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will work in small groups to determine the area of several composite figures. Students will need to apply previous knowledge to determine the areas.
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: : Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

NJ SEL Competencies

- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ
- Responsible Decision-Making: Identify the consequences associated with one's actions in order to make constructive choices.

Climate Change

- The Math Behind Sea Level Rise: In this lesson, students use principles of geometry (i.e. surface area, volume) to investigate the question "How much would sea levels rise if Antarctica melted?"
- Making Space for Food: This activity helps students visualize the area of land needed to grow or raise specific foods. While practicing geometry, the students gain some insight into the disproportionate land requirements of livestock compared to plant-based protein sources.

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can share different methods for finding the area of the same composite figure. Students can justify their area calculations in a small group setting.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:	**ELL:** ○ Create and explain orally the proofs of theorems in

○ Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook. ○ Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency. ○ Encourage students to verbalize their thinking while working in small groups by asking, assessing and advancing questions. ○ Use this information to tailor instruction to student needs. ○ Encourage students to justify their reasoning. Provide students with sentence stems if needed. ○ Anchor charts to model strategies and processes. ○ Reference sheets that list formulas, step-by-step procedures and model strategies. ○ Conceptual word wall that contains definition, translation, pictures and/or examples. ○ Graphic organizers to help students model geometric properties. ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors. ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily. ○ Videos to reinforce skills and thinking behind concepts. ○ Access to tools such as compass, protractor, ruler, graph paper and patty paper to solve problems. ○ Have students create their own composite figures to assist them in determining shapes that make up a composite figure.	a student's native language and/or use gestures, examples and selected technical words. ○ Have students work in triads or small groups where they are able to support each other's learning by giving each other input and filling in gaps in background. Students often work best when they have defined roles (surrounding the content they are studying) that they are responsible for. ○ Supply a translation dictionary. ○ Use sentence stems to provide additional language support. ○ Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills. ○ The students can explain through a "think aloud" and demonstrate how they solved the problem or explain their proof work. ○ Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit). ○ Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Use anchor charts with important terms, problem solving approaches, pictures and translations as

○ Demonstrate that three cones will fill a cylinder of the same height and radius. Use this to determine the formula for volume of a cone. Repeat process for pyramids and rectangular prisms.	needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Prompt students to use an organizational chart to answer multi-step or multi-part word problems.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can extend their knowledge of similar figures and composite areas to determine the areas of composite similar figures (11.4 Enrichment: Areas of Similar Figures)
  - ⇒ Students can explore and discover a formula for determining the area of any regular polygon (10.4 Enrichment: Formulas for Regular Polygons)
  - ⇒ Students can calculate the perimeter and area of regular polygons on the coordinate plane (11.4 Geometry Lab: Regular Polygons on the Coordinate Plane).
  - ⇒ Students can use a circle and a protractor to create regular polygons and then calculate their areas and perimeters (11.4 Enrichment: Areas of Inscribed Polygons).
  - ⇒ Students will apply concepts of visualizing relationships between two-dimensional and three-dimensional objects by interpreting and drawing topographic maps (Lab 12.1).
  - ⇒ Students can use volume and surface area formulas to minimize costs of a product (12.2 Enrichment: Minimizing Costs in Manufacturing).
  - ⇒ Students can use patterns to make cones and then determine their surface area. They can use this process to inductively determine a formula to calculate the surface area of any cone (12.3 Enrichment: Cone Patterns).
  - ⇒ Students can calculate the density of polyhedrons and spheres (12.6 Enrichment: Spheres and Density)
  - ⇒ Students can investigate how changes in dimension affect the surface area and volume of a rectangular prism (12.4 Graphing Technology Lab: Changing Dimensions).

Appendix A: Culturally Relevant Pedagogy Examples

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION	KNOWLEDGE CONSTRUCTION	PREJUDICE REDUCTION	EQUITABLE PEDAGOGY	EMPOWERING SCHOOL CULTURE
Teachers use examples and content from a variety of cultures & groups.	Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.	Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.	Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.	Using the other four dimensions to create a safe and healthy educational environment for all.
This unit / lesson is connected to other topics explored with students. There are multiple viewpoints reflected in the content of this unit / lesson. The materials and resources are reflective of the diverse identities and experiences of students. The content affirms students, as well as exposes them to experiences other than their own.	This unit / lesson provides context to the history of privilege and oppression. This unit / lesson addresses power relationships. This unit / lesson help students to develop research and critical thinking skills. This curriculum creates windows and mirrors* for students.	This unit / lesson help students question and unpack biases & stereotypes. This unit / lesson help students examine, research and question information and sources. The curriculum encourage discussion and understanding about the groups of people being represented. This unit / lesson challenges dominant perspectives.	The instruction has been modified to meet the needs of each student. Students feel respected and their cultural identities are valued. Additional supports have been provided for students to become successful and independent learners. Opportunities are provided for student to reflect on their learning and provide feedback.	There are opportunities for students to connect with the community. My classroom is welcoming and supportive for all students? I am aware of and sensitive to the needs of my students and their families. There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition), New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

Appendix C: WIDA ELD Standards Integration

ELD-MA.9-12 Explain Interpretive	Interpret mathematical explanations by • Identifying concept or entity • Analyzing data and owning problem-solving approaches • Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles
ELD-MA 9-12 Explain Expressive	Construct mathematical explanations that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions
ELD-MA.9-12 Argue Interpretive	Interpret mathematics arguments by • Comparing conjectures with previously established results and stated assumptions • Distinguishing correct from flawed logic • Evaluating relationships among evidence and mathematical principles to create generalizations
ELD-MA.9-12 Argue Expressive	Construct mathematics arguments that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions

Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is ...	**Enrichment is not...**
• Planned and purposeful • Different, or differentiated, work – not just more work • Responsive to students’ needs and situations • A promotion of high-level thinking skills and making connections within content • The ability to apply different or multiple strategies to the content • The ability to synthesize concepts and make real world and cross curricular connections • Elevated contextual complexity • Sometimes independent activities, sometimes direct instruction • Inquiry based or open-ended assignments and projects • Using supplementary materials in addition to the normal range of resources • Choices for students • Tiered/Multi-level activities with flexible groups (may change daily or weekly)	• Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) • Worksheets that are more of the same (busywork) • Random assignments, games, or puzzles not connected to the content areas or areas of student interest • Extra homework • A package that is the same for everyone • Thinking skills taught in isolation • Unstructured free time

Appendix F: Resources

Textbook: Carter, John A. Geometry. Columbus, OH: Glencoe/McGraw-Hill, 2010 & 2012

POR Text: Bass & Johnson, Geometry. Upper Saddle River, NJ: Prentice Hall: Pearson, 2011

Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.